

**Birzeit University**  
**Mathematics Department**  
 Math 132 - First Exam  
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Question 1. (36%). Circle the most correct answer:

1.  $\int \frac{dx}{x(\ln x)^2} =$

- (a)  $\ln\left(\frac{1}{x}\right) + c.$
- (b)  $\frac{\ln x}{x} + c.$
- (c)  $\frac{x}{\ln x} + c.$
- (d)  $\frac{-1}{\ln x} + c.$

$\ln x = u \Rightarrow \frac{1}{x} dx = du$   
 $\int \frac{dx}{x(\ln x)^2} = \int \frac{du}{u^2} = \int u^{-2} du = \frac{u^{-2+1}}{-2+1} = \frac{u^{-1}}{-1} = -\frac{1}{u} = -\frac{1}{\ln x} + c$

2.  $\int \frac{2x+(x+5)^{1/3}}{x+5} dx =$

- (a)  $2(x+5) + 10 \ln|x+5| - 3\sqrt[3]{x+5} + c.$
- (b)  $2(x+5) + 10e^{(x+5)} + 3\sqrt[3]{x+5} + c.$
- (c)  $2(x+5) - 10 \ln|x+5| + 3\sqrt[3]{x+5} + c.$
- (d)  $2(x+5) - 10e^{(x+5)} - 3\sqrt[3]{x+5} + c.$

$\int \frac{2x+(x+5)^{1/3}}{x+5} dx = \int \frac{2x}{x+5} + \int \frac{(x+5)^{1/3}}{x+5}$   
 $= \int \frac{2(x+5)-10}{x+5} + \int (x+5)^{-2/3}$   
 $= 2 \int \frac{x+5}{x+5} - 10 \int \frac{1}{x+5} + \int (x+5)^{-2/3}$   
 $= 2(x+5) - 10 \ln|x+5| + \int (x+5)^{-2/3}$   
 $= 2(x+5) - 10 \ln|x+5| + \frac{(x+5)^{-2/3+1}}{-2/3+1} = 2(x+5) - 10 \ln|x+5| + 3(x+5)^{1/3} + c$

3. If  $\cosh x = \frac{5}{4}$ ,  $x < 0$  then  $\sinh x$

- (a)  $\frac{3}{4}$ .
- (b)  $\frac{-4}{3}$ .
- (c)  $\frac{4}{3}$ .
- (d)  $\frac{-3}{4}$ .

$\cosh^2 x - \sinh^2 x = 1$   
 $\left(\frac{5}{4}\right)^2 - \sinh^2 x = 1$   
 $\frac{25}{16} - \sinh^2 x = 1$   
 $-\sinh^2 x = 1 - \frac{25}{16} = \frac{16-25}{16} = \frac{-9}{16}$   
 $\sinh^2 x = \frac{9}{16}$   
 $\sinh x = \pm \sqrt{\frac{9}{16}} = \pm \frac{3}{4}$   
 Since  $x < 0$ ,  $\sinh x < 0$ , so  $\sinh x = -\frac{3}{4}$ .

4.  $\int \frac{dx}{x^5-1}$  is

- (a) converges.
- (b) diverges.

$\frac{1}{x^5-1} = \frac{1}{x^5} + \frac{1}{x^4} + \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} + \dots$   
 $\int \frac{1}{x^5} = \frac{x^{-4}}{-4} = -\frac{1}{4x^4}$   
 $\frac{1}{x^5} > \frac{1}{x^5-1}$   
 $\frac{1}{x^5} > \frac{1}{x^5-1} > \frac{1}{x^5}$  for  $0 < x < 1 \Rightarrow \infty$

$3(1+4x^2)$

$\frac{1}{1+4x^2} \cdot \frac{1}{3}$

~~$\frac{1}{1+4x^2} \cdot \frac{1}{3}$~~

- $\int x^2 \tan^{-1}(2x) dx = - \int \frac{x^2}{1+4x^2}$
- (a)  $\frac{x^3}{3} \tan^{-1}(2x) - \frac{1}{48} \ln(1+4x^2) + \frac{x^2}{12} + c$
  - (b)  $\frac{x^3}{3} \tan^{-1}(2x) + \frac{1}{48} \ln(1+4x^2) + \frac{x^2}{12} + c$
  - (c)  $\frac{x^3}{3} \tan^{-1}(2x) + \frac{1}{48} \ln(1+4x^2) - \frac{x^2}{12} + c$
  - (d)  $\frac{x^3}{3} \tan^{-1}(2x) - \frac{1}{48} \ln(1+4x^2) - \frac{x^2}{12} + c$

$\frac{1}{4} \tan^{-1} \dots$   
 $(\frac{1}{4} + \frac{1}{4} \tan^{-1}) = \sec^{-1}$   
 $\cos^2 x = \frac{\cos^2 x + 1}{2}$   
 $\int \frac{x^2 \sec^2 x \cos 2x}{4(\sec^2 x)} = \frac{1}{4} \int x^2 \sec^2 x \cos 2x$   
 $2x = \dots$

10.  $\int \sin x \cos^2(2x) dx =$
- (a)  $\frac{4}{3} \cos^3 x - \frac{4}{5} \cos^5 x - \cos x + c$
  - (b)  $\frac{4}{3} \sin^3 x - \frac{4}{5} \sin^5 x - \sin x + c$
  - (c)  $\frac{4}{3} \sin^3 x - \frac{4}{5} \cos^5 x + \sin x + c$
  - (d)  $\frac{4}{3} \cos^3 x - \frac{4}{5} \sin^5 x + \cos x + c$

$\sin x \cos 2x \cos 2x$   
 $\int \sin x (2\cos^2 x - 1)(2\cos^2 x - 1)$   
 $= \int \sin x (4\cos^4 x - 4\cos^2 x + 1)$   
 $u = \cos x \Rightarrow du = -\sin x$   
 $\Rightarrow - \int (4u^4 - 4u^2 + 1) du$   
 $= -\frac{4u^5}{5} + \frac{4u^3}{3} - u$   
 $= -\frac{4\cos^5 x}{5} + \frac{4\cos^3 x}{3} - \cos x + c$

- $\int \frac{dx}{(1-9x^2)^{3/2}}$
- (a)  $\frac{x}{\sqrt{1-9x^2}} + c$
  - (b)  $\frac{x}{3\sqrt{1-9x^2}} + c$
  - (c)  $\frac{3x}{\sqrt{1-9x^2}} + c$
  - (d)  $\frac{\sqrt{1-9x^2}}{3x} + c$

$\frac{1}{1-9x^2} = \frac{1}{(1-3x)(1+3x)}$   
 $\frac{1}{(1-3x)(1+3x)} = \frac{A}{1-3x} + \frac{B}{1+3x}$   
 $1 = A(1+3x) + B(1-3x)$   
 $1 = A + 3Ax + B - 3Bx$   
 $1 = (A+B) + (3A-3B)x$   
 $A+B=1$   
 $3A-3B=0 \Rightarrow A=B$   
 $2A=1 \Rightarrow A=B=\frac{1}{2}$   
 $\int \frac{1}{(1-9x^2)^{3/2}} = \frac{1}{2} \int \frac{1}{1-3x} + \frac{1}{2} \int \frac{1}{1+3x}$   
 $= \frac{1}{2} (-\frac{1}{3} \ln|1-3x| + \frac{1}{3} \ln|1+3x|) + c$   
 $= \frac{1}{6} \ln \left| \frac{1+3x}{1-3x} \right| + c$

12. Solve for x:  $4^{(\log_2 x)} - 3e^{(\ln x)} = 10^{(\log 4)}$
- (a)  $x = 4, x = -1$
  - (b)  $x = -1$
  - (c)  $x = -4, x = 1$
  - (d)  $x = 4$

$2^{2 \log_2 x} - 3x = 10^{(\log 4)}$   
 $\Downarrow$   
 $x^2 - 3x = 4$   
 $x^2 - 3x - 4 = 0$   
 $(x-4)(x+1) = 0$   
 $x = 4, x = -1$

$\frac{dx}{(1-9x^2)^{3/2}}$   
 $u = \frac{1}{\sqrt{1-9x^2}}$   
 $du = \frac{1}{2} (1-9x^2)^{-3/2} \cdot (-18x) dx$   
 $du = -9x (1-9x^2)^{-3/2} dx$   
 $\frac{1}{9} du = -x (1-9x^2)^{-3/2} dx$   
 $\frac{1}{9} du = -\frac{1}{9} \frac{du}{u^2}$   
 $du = -du$   
 $2u = \frac{2}{\sqrt{1-9x^2}}$   
 $\frac{2}{\sqrt{1-9x^2}} = \frac{2}{3} \ln \left| \frac{1+3x}{1-3x} \right| + c$

$\frac{dx}{(1-9x^2)^{3/2}}$   
 $\frac{1}{(1-9x^2)^{3/2}} = \frac{1}{(1-3x)(1+3x)^{3/2}}$   
 $\frac{1}{(1-3x)(1+3x)^{3/2}} = \frac{A}{1-3x} + \frac{B}{1+3x} + \frac{C}{(1+3x)^{3/2}}$   
 $1 = A(1+3x)^{3/2} + B(1-3x)(1+3x)^{3/2} + C(1-3x)$   
 $1 = A(1+3x)^{3/2} + B(1-9x^2)(1+3x)^{3/2} + C(1-3x)$   
 $1 = A(1+3x)^{3/2} + B(1-9x^2)(1+3x)^{3/2} + C(1-3x)$   
 $1 = A(1+3x)^{3/2} + B(1-9x^2)(1+3x)^{3/2} + C(1-3x)$

Question 2. (10%) Find the length of the curve  $x = \left(\frac{y}{8}\right)^2 - 2 \ln \frac{y}{4}$ ,  $4 \leq y \leq 12$ .  
 (DO NOT EVALUATE THE INTEGRAL)

$$x' = \frac{y}{8} - \frac{2}{y}$$

$$x'^2 = \left(\frac{y}{8} - \frac{2}{y}\right)^2$$

$$x'^2 + 1 = \left(\frac{y}{8} - \frac{2}{y}\right)^2 + 1$$

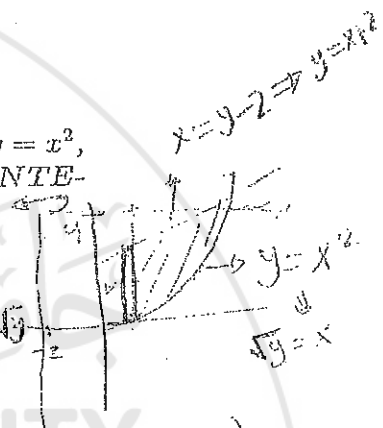
$$L = \int \sqrt{\left(\frac{y}{8} - \frac{2}{y}\right)^2 + 1} dy$$

Question 3. (10%) Find the volume generated by revolving the area between  $y = x^2$ ,  $x = y - 2$ , in the first quadrant about the y-axis. (DO NOT EVALUATE THE INTEGRALS)

1.  $x = -2$  using Shell method

$$V = 2\pi \int (\text{shell radius})(\text{shell height}) dy$$

$$V = 2\pi \int (-y+2)(y-2)\sqrt{y} dy$$



2.  $y = 4$  using Washer method

$$V = \pi \int R^2 - r^2 dx \quad R = 4 - x^2, \quad r = 4 - (x+2) = 2 - x$$

$$\Rightarrow V = \pi \int (4-x^2)^2 - (2-x)^2 dx = \pi \int 2 - x^2 + x dx$$

Question 4. (16%) Determine whether the following integrals converge or diverge:

$$\int_0^{\infty} \frac{dx}{\sqrt{x^6+1}}$$

$$\int_0^{\infty} \frac{dx}{\sqrt{x^6+1}} \sim \int_0^{\infty} \frac{dx}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{1}{2x^2} = 0$$

$$\lim_{x \rightarrow \infty} \frac{1}{2x^2} = \frac{1}{2}$$

$\int_0^{\infty} \frac{dx}{\sqrt{x^6+1}} = \int_0^1 \frac{dx}{\sqrt{x^6+1}} + \int_1^{\infty} \frac{dx}{\sqrt{x^6+1}}$   
 by D.C.T with  $\frac{1}{x^3}$  is converge  
 $\lim_{x \rightarrow \infty} \frac{1/x^3}{1/\sqrt{x^6+1}} = \lim_{x \rightarrow \infty} \frac{1}{x^3} = 0$   
 $\int_1^{\infty} \frac{1}{x^p} dx$   $0 < p < 1 \Rightarrow \infty$  therefore  
 $\int_0^{\infty} \frac{dx}{\sqrt{x^6+1}} \rightarrow \infty$   
 So  $\int_0^{\infty} \frac{dx}{\sqrt{x^6+1}}$  is diverge.

$$\int_0^{\infty} \frac{dx}{x^3+x^2}$$

$$\int_0^{\infty} \frac{dx}{x^3+x^2} = \int_0^1 \frac{dx}{x^3+x^2} + \int_1^{\infty} \frac{dx}{x^3+x^2}$$

$$\int_0^1 \frac{dx}{x^3+x^2} + \int_1^{\infty} \frac{dx}{x^3+x^2}$$

$\int_0^1 \frac{1}{x^p} dx$   $0 < p < 1$   
 $\Rightarrow$  is diverge  
 therefore  $\int_0^{\infty} \frac{dx}{x^3+x^2}$  is diverge

by D.C.T with  $\frac{1}{x^3}$   
 $\int_1^{\infty} \frac{dx}{x^3+x^2}$  is converge  
 $\Rightarrow \int_0^{\infty} \frac{dx}{x^3+x^2}$  is diverge.

5. If  $f(x) = \sinh x$ ,  $g(x) = e^x$ , then as  $x$  approaches infinity

- (a)  $f(x)$  grows faster than  $g(x)$ .
- (b)  $f(x)$  grows slower than  $g(x)$ .
- (c)  $f(x)$  and  $g(x)$  grow at the same rate.
- (d) none of the above.

$$\frac{e^x - e^{-x}}{2} = \frac{1}{2} \left( \frac{e^x \cdot e^{-x}}{e^x} \right)$$

$$\rightarrow \frac{1}{2} \left( \frac{1 - e^{-2x}}{e^x} \right)$$

6.  $\int_0^{\ln 2} e^{2x} \cosh x dx =$

- (a)  $\frac{2}{3}$
- (b)  $\frac{5}{3}$
- (c)  $\frac{3}{5}$
- (d)  $\frac{3}{2}$

$$e^{2x} \cosh x = \frac{e^{2x}(e^x + e^{-x})}{2}$$

$$\int \frac{e^{3x} + e^x}{2} dx = \frac{1}{2} \left( \frac{e^{3x}}{3} + e^x \right)$$

$$\left[ \frac{e^{3x}}{6} + \frac{e^x}{2} \right]_0^{\ln 2} = \left( \frac{8}{6} + 1 \right) - \left( \frac{1}{6} + \frac{1}{2} \right) = \frac{14}{6} - \frac{4}{6} = \frac{10}{6} = \frac{5}{3}$$

7. The volume of the solid whose cross sections are circular disks whose diameters run from  $y = x^2$  to  $y = x$ ,  $0 \leq x \leq 1$

- (a)  $\frac{\pi}{120}$
- (b)  $\frac{\pi}{30}$
- (c)  $\frac{\pi}{40}$
- (d)  $\frac{\pi}{60}$

SA =  $\int \pi r^2 dx$

$$\int_0^1 \pi \left( \frac{x-x^2}{2} \right)^2 dx = \frac{\pi}{4} \int_0^1 (x-x^2)^2 dx$$

$$= \frac{\pi}{4} \int_0^1 (x^2 - 2x^3 + x^4) dx = \frac{\pi}{4} \left( \frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big|_0^1$$

$$= \frac{\pi}{4} \left( \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) = \frac{\pi}{4} \left( \frac{10 - 15 + 6}{30} \right) = \frac{\pi}{4} \left( \frac{1}{30} \right) = \frac{\pi}{120}$$

Solve the differential equation:  $y' = \cot x \ln(\sin x)$ ;  $y(\frac{\pi}{2}) = 0$

- (a)  $\frac{(\ln(\cos x))^2}{2} + 2$
- (b)  $\frac{(\ln(\sin x))^2}{2}$
- (c)  $\ln(\sin x)$
- (d)  $\frac{(\ln(\sin x))^2}{2} + 1$

~~Let  $u = \ln(\sin x)$~~

$$u = \sin x$$

$$du = \cos x dx$$

$$dx = \frac{du}{\cos x}$$

$$\int \cot x \ln(\sin x) dx = \int \frac{\ln u}{u} du = \frac{1}{2} (\ln u)^2 + C$$

$$= \frac{1}{2} (\ln(\sin x))^2 + C$$

$\Rightarrow \frac{1}{2} (\ln(\sin x))^2$

Let  $u = x^2 - x^3 + x^4$

$$\frac{d}{dx} (x^2 - x^3 + x^4) = 2x - 3x^2 + 4x^3$$

$$\int (2x - 3x^2 + 4x^3) dx = x^2 - x^3 + x^4 + C$$

$$\frac{d}{dx} e^{-x} = -e^{-x}$$

Question 5. (28%) Evaluate:

1.  $\int \frac{e^{2x}}{e^{2x}+1} dx$

$$\Rightarrow \int \frac{e^{2x}}{e^{2x}(1+\frac{1}{e^x})} dx \Rightarrow \int \frac{e^x}{1+e^x} dx \Rightarrow \int \frac{-e^{-x}}{-1+e^{-x}} dx = -\ln|1+e^{-x}|$$

2.  $\int \frac{x^2+1}{x^2-x} dx$

$$\Rightarrow \int \frac{1+x}{x(x-1)} dx \Rightarrow \int \frac{1}{x} dx + \int \frac{1+x}{x(x-1)} dx$$

$$\int \frac{1+x}{x(x-1)} dx = \int \frac{A}{x} dx + \int \frac{B}{x-1} dx \quad \begin{matrix} -1+B=1 \\ +1 \end{matrix}$$

$$\Rightarrow 1+x = Ax - A + Bx \Rightarrow Ax + Bx = x \rightarrow \textcircled{1}$$

$$\Rightarrow A = 1 \text{ \& } B = 2 \quad -A = 1 \rightarrow \textcircled{2}$$

$$\Rightarrow \int \frac{1}{x} dx + \int \frac{2}{x-1} dx \Rightarrow -\ln|x| + 2\ln|x-1|$$

$$\Rightarrow \text{Final answer: } x + -\ln|x| + 2\ln|x-1|$$

3.  $\int \frac{dx}{\sqrt{1-x}}$

$$\int \frac{dx}{\sqrt{1-x}} \Rightarrow u=1-x \quad du=-dx$$

$$\Rightarrow \int \frac{dx}{\sqrt{1-x}} \Rightarrow u=1-x \quad du=-dx$$

$$\Rightarrow \int \frac{-du}{\sqrt{u}} = -2\sqrt{u} = -2\sqrt{1-x}$$

$$\Rightarrow -2\sqrt{u}$$

$$\lim_{b \rightarrow -1^+} -2 + 2\sqrt{b} = 1$$

$$\int_0^1 \frac{dx}{\sqrt{1-x}} + \int_0^1 \frac{dx}{\sqrt{1-x}}$$

$$\Rightarrow -2\sqrt{u} + -2\sqrt{u}$$

$$\lim_{b \rightarrow -1^+} +2\sqrt{b} + -2 = \text{diverge}$$