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Birzeit University
Mathematics Department
Math 132 - First Exam

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StudentName: [redacted] .... [redacted] ......Number: [redacted]

Instructors:

a.Iflaifel Majed   b.Maher Abdullatef   c.We'am Abu Arqoub

Question 1. (36%).Circle the most correct answer:

1. $\int \frac{dx}{x(\ln x)^2} =$

   (a) $\ln(\frac{1}{x}) + c.$
   (b) $\frac{\ln x}{x} + c.$
   (c) $\frac{x}{\ln x} + c.$
   (d) $\frac{-1}{\ln x} + c.$

   $\ln x = u$, $\frac{dx}{x} = du$, $dx = x\,du \Rightarrow \int \frac{x\,du}{x(u)^2} \Rightarrow \int u^{-2}du$, $\frac{u^{-2+1}}{-2+1}$, $-\frac{1}{u}$, $= -\frac{1}{u} \Rightarrow \frac{-1}{\ln x} + c$

2. $\int \frac{2x+(x+5)^{\frac{1}{3}}}{x+5} dx =$

   (a) $2(x+5) + 10\ln|x+5| - 3\sqrt[3]{x+5} + c.$
   (b) $2(x+5) + 10e^{(x+5)} + 3\sqrt[3]{x+5} + c.$
   (c) $2(x+5) - 10\ln|x+5| + 3\sqrt[3]{x+5} + c.$
   (d) $2(x+5) - 10e^{(x+5)} - 3\sqrt[3]{x+5} + c.$

   $\Rightarrow \int \frac{2x}{x+5} + \int \frac{(x+5)^{1/3}}{x+5} \Rightarrow \int (x+5)^{-2/3} \Rightarrow u = x+5$, $du = 1 \Rightarrow \int (u)^{-2/3+1}$, $\frac{3u^{1/3}}{}$, $3\sqrt[3]{x+5}$

   $\Rightarrow \int 2 + \frac{-10}{x+5}dx$, $-10\ln|x+5| + 3\sqrt[3]{x+5}$, $\Rightarrow 2x +$

3. If $\cosh x = \frac{5}{4}$, $x < 0$ then $\sinh x$

   (a) $\frac{3}{4}.$
   (b) $\frac{-4}{3}.$
   (c) $\frac{4}{3}.$
   (d) $\frac{-3}{4}.$

   $\cosh^2 x - \sinh^2 x = 1$, $\frac{25}{16} - \sinh^2 x = 1$, $\frac{25}{16} - \frac{16}{16} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

4. $\int_0^1 \frac{dx}{x^5 - 1}$ is

   (a) converges.
   (b) diverges.

   $\int x^{-5} = -\frac{x^{-4}}{4}$, $\frac{1}{x^5 - 1}$, $\frac{1}{x^5}$, $0 < P < 1 \Rightarrow \infty$, $\frac{\frac{1}{x^5}}{x^5 - 1}$

$\int x^2 \tan^{-1}(2x)dx =$

(a) $\frac{x^3}{3}\tan^{-1}(2x) - \frac{1}{48}\ln(1+4x^2) + \frac{x^2}{12} + c.$

(b) $\frac{x^3}{3}\tan^{-1}(2x) + \frac{1}{48}\ln(1+4x^2) + \frac{x^2}{12} + c.$

(c) $\frac{x^3}{3}\tan^{-1}(2x) + \frac{1}{48}\ln(1+4x^2) - \frac{x^2}{12} + c.$

(d) $\frac{x^3}{3}\tan^{-1}(2x) - \frac{1}{48}\ln(1+4x^2) - \frac{x^2}{12} + c.$

10. $\int \sin x \cos^2(2x)dx =$

(a) $\frac{4}{3}\cos^3 x - \frac{4}{5}\cos^5 x - \cos x + c.$

(b) $\frac{4}{3}\sin^3 x - \frac{4}{5}\sin^5 x - \sin x + c.$

(c) $\frac{4}{3}\sin^3 x - \frac{4}{5}\cos^5 x + \sin x + c.$

(d) $\frac{4}{3}\cos^3 x - \frac{4}{5}\sin^5 x + \cos x + c.$

$\int \frac{dx}{(1-9x^2)^{(\frac{3}{2})}}$

(a) $\frac{x}{\sqrt{1-9x^2}} + c.$

(b) $\frac{x}{3\sqrt{1-9x^2}} + c.$

(c) $\frac{3x}{\sqrt{1-9x^2}} + c.$

(d) $\frac{\sqrt{1-9x^2}}{3x} + c.$

12. Solve for x: $4^{(\log_2 x)} - 3e^{(\ln x)} = 10^{(\log 4)}$

(a) $x = 4$, $x = -1$.

(b) $x = -1$.

(c) $x = -4$, $x = 1$.

(d) $x = 4$.

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Question 2. *(10%) Find the length of the curve* $x = (\frac{y}{4})^2 - 2\ln\frac{y}{4}$, $4 \le y \le 12$. *(DO NOT EVALUATE THE INTEGRAL)*

$x' = \frac{y}{8} - \frac{2}{y}$

$x'^2 = \left(\frac{y}{8} - \frac{2}{y}\right)^2$

$x'^2 + 1 = \left(\frac{y}{8} - \frac{2}{y}\right)^2 + 1$

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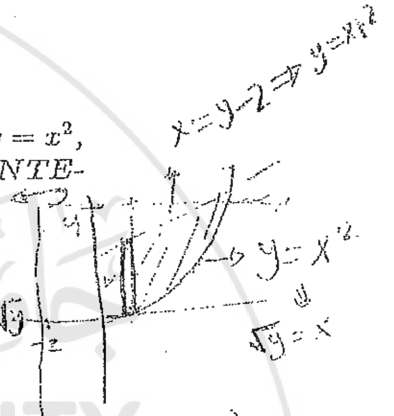
$L = \int \sqrt{\left(\frac{y}{8} - \frac{2}{y}\right)^2 + 1}\, dy$

Question 3. *(10%) Find the volume generated by revolving the area between* $y = x^2$, $x = y - 2$, *in the first quadrant about. (DO NOT EVALUATE THE INTEGRALS)*



1. $x = -2$ *using Shell method*

$V = 2\pi \int (\text{shell radius})(\text{shell height}) \Rightarrow R = \sqrt{y}+2$

$h = (y-2) - \sqrt{y}$

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$V = 2\pi \int (\sqrt{y}+2)(y-2)\sqrt{y}\, dy$

2. $y = 4$ *using Washer method*

$(y-x^2) =$

$y - x^2 - 2 + x$

$V = \pi \int R^2 - r^2 \quad R = 4 - x^2,\ r = 4 - (x+2) = 2 - x$

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$\Rightarrow V = \pi \int (4-x^2)^2 (2-x)^2\, dx = \pi \int 2 - x^2 + x\, dx.$

$\frac{1}{2x^2}$

Question 4. *(16%) Determine wether the following integrals converge or diverge:*

a. $\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$

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$\Rightarrow \int \frac{dx}{\sqrt{x^6+1}} \le \int \frac{dx}{x^3}$

$\lim_{x\to 0^+} \frac{-1}{2x^2} = -\infty \quad \lim_{x\to\infty} \frac{-1}{2x^2} = 0$

$\lim_{x\to 1} \frac{-1}{2x^2} = \frac{-1}{2}$

$\Rightarrow \int_0^1 \frac{dx}{\sqrt{x^6+1}} + \int_1^\infty \frac{dx}{\sqrt{x^6+1}} \rightarrow$ by D.C.T with $\frac{1}{x^3}$ $\int_1^\infty \frac{dx}{\sqrt{x^6+1}}$ is converge

$\lim_{x\to\infty} \frac{\frac{1}{x^3}}{\frac{1}{\sqrt{x^6+1}}} = \lim_{x\to\infty}$

$\int_0^1 \frac{1}{x^p}\ 0<p<1 \Rightarrow \infty$ therefore $\int_0^1 \frac{dx}{\sqrt{x^6+1}} \rightarrow \infty$

So $\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$ is diverge.

b. $\int_0^\infty \frac{dx}{x^3 + x^{\frac{2}{3}}}$

$\frac{1}{x^3}$

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$\int_0^1 \frac{dx}{x^3 + x^{2/3}} + \int_1^\infty \frac{dx}{x^3+x^{2/3}}$ by D.C.T with $\frac{1}{x^3}$ $\int_1^\infty \frac{dx}{x^3+x^{2/3}}$ is converge

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$\int_0^1 \frac{1}{x^p}\ 0<p<1$

$\Rightarrow$ diverge, Therefore $\int_0^1 \frac{dx}{x^3+x^{2/3}}$ is divege $\Rightarrow \int_0^\infty \frac{dx}{x^3+x^{2/3}}$ is diverge.

5. If $f(x) = \sinh x$, $g(x) = e^x$, then as x approaches infinity

(a) $f(x)$ grows faster than $g(x)$.

(b) $f(x)$ grows slower than $g(x)$.

(c) $f(x)$ and $g(x)$ grow at the same rate.

(d) none of the above.

6. $\int_0^{\ln 2} e^{(2x)} \cosh x dx =$

(a) $\frac{2}{3}$.

(b) $\frac{5}{3}$.

(c) $\frac{3}{5}$.

(d) $\frac{3}{2}$.

7. The volume of the solid whose cross sections are circular disks whose diameters run from $y = x^2$ to $y = x$, $0 \le x \le 1$

(a) $\frac{\pi}{120}$.

(b) $\frac{\pi}{30}$.

(c) $\frac{\pi}{40}$.

(d) $\frac{\pi}{60}$.

8. Solve the differential equation: $y' = \cot x \ln(sinx)$, $y(\frac{\pi}{2}) = 0$

(a) $\frac{(\ln(\cos x))^2}{2} + 2$.

(b) $\frac{(\ln(\sin x))^2}{2}$.

(c) $\ln(\sin x)$.

(d) $\frac{(\ln(\sin x))^2}{2} + 1$.

$\frac{\pi}{4}\int (x-x^2)^2 = \frac{\pi}{4}\int x^4 - 2x^3 + x^2$

$\frac{\pi}{4}\left(\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3}\right)$

$\frac{\pi}{120}$

$\frac{d}{dx} e^{-x} = -\frac{e^{-x}}{5}$

Question 5. *(28%) Evaluate:*

1. $\int \frac{e^{2x}}{e^x+1}\,dx$

$\Rightarrow \int \frac{e^{2x}}{e^x(1+\frac{1}{e^x})} \Rightarrow \int \frac{e^x}{1+\frac{1}{e^x}} \Rightarrow -\int \frac{-e^{-x}}{1+e^{-x}} = -\ln|1+e^{-x}|$

$u =$

2. $\int \frac{x^2+1}{x^2-x}\,dx$

$\Rightarrow \int 1 + \frac{1+x}{x^2-x}\,dx \Rightarrow \int 1\,dx + \int \frac{1+x}{x(x-1)}\,dx$

$\Rightarrow \int \frac{1+x}{x(x-1)}\,dx = \int \frac{A}{x}\,dx + \int \frac{B}{(x-1)}\,dx$ $\quad -1+B=1$

$\Rightarrow 1+x = Ax - A + Bx \Rightarrow Ax+Bx = x \rightarrow ①$

$-A = 1 \rightarrow ②$

$\Rightarrow A=-1 \;\&\; B=2$

$\Rightarrow \int \frac{-1}{x}\,dx + \int \frac{2}{x-1}\,dx \Rightarrow -\ln|x| + 2\ln|x-1|$

$\Rightarrow$ Final answer : $x + -\ln|x| + 2\ln|x-1|$

3. $\int_{-1}^{1} \frac{dx}{\sqrt{1-x}}$

$\int \frac{dx}{\sqrt{1-x}} \Rightarrow u = 1-x,\; du = -dx$

$\Rightarrow \int -\frac{du}{\sqrt{u}} = -2\sqrt{u}\Big|_{-1}^{b}$

$\Rightarrow -2 + 2\sqrt{b}$

$\lim_{b\to -1^+} -2 + 2\sqrt{b} =$

$\Rightarrow \int \frac{dx}{\sqrt{1-x}} \Rightarrow u = 1-x,\; du = -dx$

$\Rightarrow -2\sqrt{u}$

$\int_{-\infty}^{0} \frac{dx}{\sqrt{1-x}} + \int_{0}^{1} \frac{dx}{\sqrt{1-x}}$

$\Rightarrow -2\sqrt{u}\Big|_{-1}^{0} + -2\sqrt{u}\Big|_{0}^{1}$

$\Rightarrow +2\sqrt{b} + -2$

$\lim_{b\to -1^+} +2\sqrt{b} + -2 =$ diverged

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